Circulating Current States in Bilayer Fermionic and Bosonic Systems

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It is shown that fermionic polar molecules or atoms in a bilayer optical lattice can undergo the transition to a state with circulating currents, which spontaneously breaks the time reversal symmetry. Estimates of relevant temperature scales are given and experimental signatures of the circulating current phase are identified. Related phenomena in bosonic and spin systems with ring exchange are discussed.

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Introduction.– The technique of ultracold gases loaded into optical lattices [1, 2] allows a direct experimental study of paradigmatic models of strongly correlated systems. The possibility of unprecedented control over the model parameters has opened wide perspectives for the study of quantum phase transitions. Detection of the Mott insulator to superfluid transition in bosonic atomic gases [3, 4, 5], of superfluidity [6, 7] and Fermi liquid [8] in cold Fermi gases, realization of Fermi systems with low dimensionality [9, 10] mark some of the recent achievements in this rapidly developing field [11]. While the atomic interactions can be treated as contact ones for most purposes, polar molecules [12, 13, 14] could provide further opportunities of controlling longer-range interactions.

In this Letter, I propose several models on a bilayer optical lattice which exhibit a phase transition into an exotic circulating current state with spontaneously broken time reversal symmetry. Those states are closely related to the "orbital antiferromagnetic states" proposed first by Halperin and Rice nearly 40 years ago [15], rediscovered two decades later [16, 17, 18] and recently found in numerical studies in extended t-J model on a ladder [19] and on a two-dimensional bilayer [20]. Our goal is to show how such states can be realized and detected in a relatively simple optical lattice setup.

Model of fermions on a bilayer optical lattice.– Consider spin-polarized fermions in a bilayer optical lattice shown in Fig. 1. The system is described by the Hamiltonian

$$\begin{aligned}\mathcal{H} &= V\sum_{\boldsymbol{r}} n_{1,\boldsymbol{r}}n_{2,\boldsymbol{r}} + \sum_{\sigma\sigma'}\sum_{\langle \boldsymbol{r}\boldsymbol{r}'\rangle} V'_{\sigma\sigma'} n_{\sigma,\boldsymbol{r}} n_{\sigma',\boldsymbol{r}'} \\ &- t\sum_{\boldsymbol{r}}(a^{\dagger}_{1,\boldsymbol{r}}a_{2,\boldsymbol{r}} + \text{h.c.}) - t'\sum_{\sigma}\sum_{\langle \boldsymbol{r}\boldsymbol{r}'\rangle}(a^{\dagger}_{\sigma,\boldsymbol{r}}a_{\sigma,\boldsymbol{r}'} + \text{h.c.})\end{aligned} \tag{1}$$

where $\boldsymbol{r}$ labels the vertical dimers arranged in a two-dimensional (2d) square lattice, $\sigma = 1, 2$ labels two layers, and $\langle \boldsymbol{r}\boldsymbol{r}'\rangle$ denotes a sum over nearest neighbors. Amplitudes t and t' describe hopping between the layers and within a layer, respectively. A strong "on-dimer" nearest-neighbor repulsion $V \gg t, t' > 0$ is assumed, and there is an interaction between the nearest-neighbor dimers $V'_{\sigma\sigma'}$ which can be of either sign.

This seemingly exotic setup can be realized by using polar molecules [13, 14], or atoms with a large dipolar magnetic moment such as $^{53}\mathrm{Cr}$ [12], and adjusting the direction of the dipoles with respect to the bilayer plane. Let θ, φ be the polar and azimuthal angles of the dipolar moment (the coordinate axes are along the basis vectors of the lattice, z axis is perpendicular to the bilayer plane). Setting $\varphi = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$ ensures the dipole-dipole interaction is the same along the x and y directions. The nearest neighbor interaction parameters in (1) take the following values: $V = (d_0^2/\ell_\perp^3)(1 - 3\cos^2\theta)$, and $V'_{12} = V'_{21} = (d_0^2/R^3)\{1-3R^{-2}(\ell_\parallel\cos\theta+\ell_\perp\sin\theta\cos\varphi)^2\}$, $V'_{11} = V'_{22} = V'_{12}(\ell_\parallel = 0)$, where d_0 is the dipole moment of the particle, $\ell_\perp$ and $\ell_\parallel$ are the lattice spacings in the directions perpendicular and parallel to the layers, respectively, and $R^2 = \ell_\parallel^2 + \ell_\perp^2$. The strength and the sign of interactions V, $\widetilde{V}'$ can be controlled by tuning the angles θ, φ and the lattice constants $\ell_\perp$, $\ell_\parallel$. Below we will see that the physics of the problem depends on the difference

$$\widetilde{V}' = V'_{11} - V'_{12}, \tag{2}$$

with the most interesting regime corresponding to $\widetilde{V}' < 0$.

Consider the model at half-filling. Since $V \gg t, t'$, we may restrict ourselves to the reduced Hilbert space containing only states with one fermion per dimer. Two states of each dimer can be identified with pseudospin-$\frac{1}{2}$ states $|{\uparrow}\rangle$ and $|{\downarrow}\rangle$. Second-order perturbation theory in t' yields the effective Hamiltonian

$$\begin{aligned}\mathcal{H}_S &= \sum_{\langle \boldsymbol{r}\boldsymbol{r}'\rangle}\left\{J(S^x_{\boldsymbol{r}}S^x_{\boldsymbol{r}'} + S^y_{\boldsymbol{r}}S^y_{\boldsymbol{r}'}) + J_z S^z_{\boldsymbol{r}}S^z_{\boldsymbol{r}'}\right\} - H\sum_{\boldsymbol{r}} S^x_{\boldsymbol{r}}, \\ J &= 4(t')^2/V, \quad J_z \equiv J\Delta = J + \widetilde{V}', \quad H = 2t,\end{aligned} \tag{3}$$

describing a 2d anisotropic Heisenberg antiferromagnet in a magnetic field perpendicular to the anisotropy axis. The twofold degenerate ground state has the Néel antiferromagnetic (AF) order transverse to the field, with spins canted towards the field direction. The AF order is along the y axis for $\Delta < 1$ (i.e., $\widetilde{V}' < 0$), and along the z axis for $\Delta > 1$ ($\widetilde{V}' > 0$).

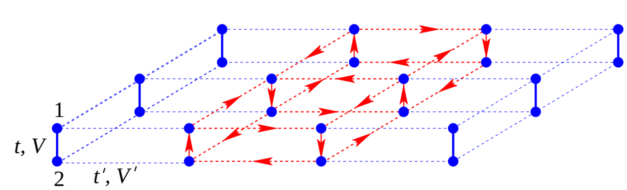

FIG. 1: Bilayer lattice model described by the Hamiltonian (1). The arrows denote particle flow in the circulating current phase.

The angle α between the spins and the field is classically given by $\cos\alpha = H/(2ZJS)$, where S is the spin value and $Z = 4$ is the lattice coordination number. This classical ground state is exact at the special point $H = 2SJ\sqrt{2(1+\Delta)}$ [21]. The transversal AF order vanishes above a certain critical field H_c; classically $H_c = 2ZJS$, and the same result follows from the spin-wave analysis of (3) (one starts with the fully polarized spin state at large H and looks when the magnon gap vanishes). This expression becomes exact at the isotropic point $\Delta = 1$ and is a good approximation for Δ close to 1.

The long-range AF order along the y direction translates in the original fermionic language into the staggered arrangement of *currents* flowing from one layer to the other:

$$N_y = (-)^r \langle S_r^y \rangle \mapsto (-)^r \langle -\frac{i}{2}(a_{1,r}^\dagger a_{2,r} - a_{2,r}^\dagger a_{1,r}) \rangle. \quad (4)$$

In terms of the original model (1), the condition $H < H_c$ for the existence of such a staggered current order becomes

$$t < 8(t')^2/V. \quad (5)$$

The continuity equation for the current and the lattice symmetry dictate the current pattern shown in Fig. 1. This circulating current (CC) state has a spontaneously broken time reversal symmetry, and is realized only for attractive inter-dimer interaction $\widetilde{V}' < 0$ (i.e., the easy-plane anisotropy $\Delta < 1$) [22]. If $\Delta = 1$, the direction of the AF order in the xy plane is arbitrary, so there is no long-range order at any finite temperature. For $\Delta > 1$ (i.e., $\widetilde{V}' > 0$) the AF order along the z axis corresponds to the density wave (DW) phase with in-layer occupation numbers having a finite staggered component.

The phase diagram in the temperature-anisotropy plane is sketched in Fig. 2. At the critical temperature $T = T_c$ the discrete Z_2 symmetry gets spontaneously broken, so the corresponding thermal phase transition belongs to the 2d Ising universality class (except the two lines $\Delta = 1$ and $H = 0$ where the symmetry is enlarged to $\mathrm{U}(1)$ and the transition becomes the Kosterlitz-Thouless one). Away from the phase boundaries the critical temperature $T_c \sim J$, but at the isotropic point $\Delta = 0$, $H = 0$ it vanishes due to divergent thermal fluctuations: for $1 - \Delta \ll 1$ and $H \ll J$, it can be estimated as

$$T_c \sim J/\ln[\min(|1-\Delta|^{-1}, J^2/H^2)]. \quad (6)$$

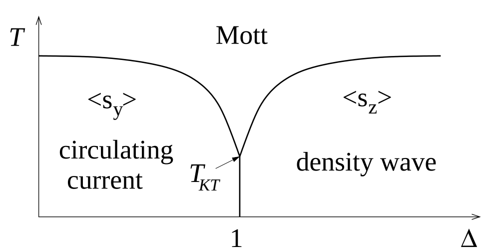

FIG. 2: Schematic phase diagram of the model (1), (3) at fixed $H = 2t$. The line $\Delta = 1$ corresponds to the Kosterlitz-Thouless phase, with the transition temperature $T_{KT} \propto J/\ln(J/H)$ at small H.

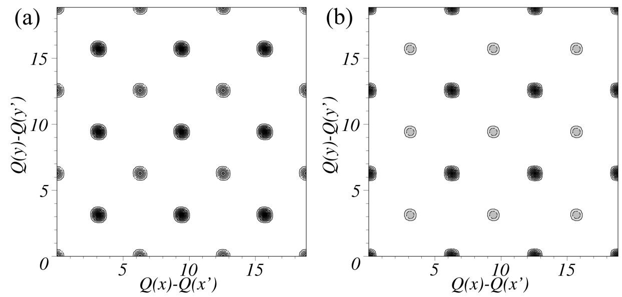

FIG. 3: Noise correlation function $\mathcal{G}(\boldsymbol{r}, \boldsymbol{r}')$ from time-of-flight images in the circulating current (CC) phase, shown as the function of the relative distance $\boldsymbol{Q}(\boldsymbol{r}) - \boldsymbol{Q}(\boldsymbol{r}')$, with $\boldsymbol{Q}(\boldsymbol{r}) = m\boldsymbol{r}/(\hbar t)$ expressed in $1/\ell_\parallel$ units: (a) $\boldsymbol{Q}(\boldsymbol{r}) = (0,0)$; (b) $\boldsymbol{Q}(\boldsymbol{r}) = \frac{\pi}{2}(1,1)$. Changing the initial point $\boldsymbol{Q}(\boldsymbol{r})$ leads to the change of relative weight of the two systems of dips, which is the fingerprint of the CC phase.

The quantum phase transition at $T = 0$, $H = H_c$ is of the 3d Ising type (except at the $\mathrm{U}(1)$-symmetric point $\Delta = 1$ where the universality class is that of the 2d dilute Bose gas [23]), so in its vicinity the CC order parameter $N_y \propto (H_c - H)^\beta$ with $\beta \simeq 0.313$ [24], and $T_c \propto JN_y^2 \propto J(H_c - H)^{2\beta}$. At $T > T_c$ or $H > H_c$ the only order parameter is $\langle S^x \rangle$, corresponding to the Mott phase with one particle per dimer.

Bilayer lattice design and hierarchy of scales.– The bilayer can be realized, e.g., by employing three pairs of mutually perpendicular counter-propagating laser beams with the same polarization and adding another pair of beams with an orthogonal polarization and additional phase shift δ, so that the resulting field intensity has the form $\big[E_\perp(\cos kx + \cos ky) + E_z \cos kz\big]^2 + \widetilde{E}_z^2 \cos^2(kz+\delta)$. Taking $\delta = \frac{\pi}{4}(1+\zeta)$ and $\widetilde{E}_z^2 > E_z(2E_\perp + \zeta E_z)$, with $\zeta = \pm 1$ for blue and red detuning, respectively, one obtains a three-dimensional stack of bilayers, separated by large potential barriers U_{3d}. Eq. (5) implies $V \gg t' \gg t, |\widetilde{V}'|$, which can be achieved by making the z-direction potential barrier $U_\perp$ inside the bilayer sufficiently larger than the in-plane barrier $U_\parallel$, so that the condition $t \ll t'$ will be met; e.g., $\widetilde{E}_z/E_\perp \approx 20$, $E_z/E_\perp \approx 15$ yields the barrier ratio $U_{3d} : U_\perp : U_\parallel$ of approximately $16 : 8 : 1$, and the lattice constants $\ell_\perp \approx 0.45\lambda$, $\ell_\parallel = \lambda$, where $\lambda = 2\pi/k$ is the laser wave length. The parameter $\widetilde{V}'$ has a zero as a function of the angle θ, so it can be made as small as needed. Taking $\lambda = 400$ nm, one obtains an estimate of $T_c = (0.1 \div 0.3)\ \mu$K for cyanide molecules ClCN and HCN with the dipolar moment $d_0 \approx 3$ Debye, while the Fermi temperature for the same parameters is $T_f \approx (0.6 \div 1.3)\ \mu$K. This estimate corresponds to the maximum value of $T_c \sim J$ reached when $\widetilde{V}' \sim -J$ and $t \lesssim J$. The hopping t' was estimated assuming the in-plane potential barrier $U_\parallel$ is roughly equal to the recoil energy $E_r = (\hbar k)^2/2m$, where m is the particle mass.

Experimental signatures. – Signatures of the ordered phases can be observed [25, 26] in time-of-flight experiments by measuring the density noise correlator $\mathcal{G}(\boldsymbol{r}, \boldsymbol{r}') =$

$\langle n(\boldsymbol{r})n(\boldsymbol{r'})\rangle - \langle n(\boldsymbol{r})\rangle\langle n(\boldsymbol{r'})\rangle$. If the imaging axis is perpendicular to the bilayer, $n(\boldsymbol{r}) = \sum_\sigma \langle a^\dagger_{\sigma,\boldsymbol{r}} a_{\sigma,\boldsymbol{r}}\rangle$ is the local net density of two layers. For large flight times t it is proportional to the momentum distribution $n_{\boldsymbol{Q}(\boldsymbol{r})}$, where $\boldsymbol{Q}(\boldsymbol{r}) = m\boldsymbol{r}/\hbar t$. In the Mott phase the response shows fermionic "Bragg dips" at reciprocal lattice vectors $\boldsymbol{g} = (2\pi h/\ell_\parallel, 2\pi k/\ell_\parallel)$,

$$\mathcal{G}_{\mathrm{M}}(\boldsymbol{r},\boldsymbol{r'}) \propto f_0(\boldsymbol{r},\boldsymbol{r'}) = -2\langle S^x\rangle^2 \sum_{\boldsymbol{g}} \delta\big(\boldsymbol{Q}(\boldsymbol{r}) - \boldsymbol{Q}(\boldsymbol{r'}) - \boldsymbol{g}\big)$$

In the CC and DW phases the noise correlator contains an additional system of dips shifted by $\boldsymbol{Q}_B = (\pi/\ell_\parallel, \pi/\ell_\parallel)$:

$$\begin{aligned}\mathcal{G}_{\mathrm{CC,DW}}(\boldsymbol{r},\boldsymbol{r'}) &\propto f_0(\boldsymbol{r},\boldsymbol{r'}) - 2\Big\{\langle S^z\rangle^2 + \langle S^y\rangle^2 \\ &\times\big[1 + \big(\cos\big(Q_x(\boldsymbol{r})\ell_\parallel\big) + \cos\big(Q_y(\boldsymbol{r})\ell_\parallel\big)\big)^2\big]\Big\} \\ &\times \sum_{\boldsymbol{g}} \delta\big(\boldsymbol{Q}(\boldsymbol{r}) - \boldsymbol{Q}(\boldsymbol{r'}) - \boldsymbol{Q}_B - \boldsymbol{g}\big) \qquad (7)\end{aligned}$$

In the DW phase $\langle S^z\rangle \neq 0$, $\langle S^y\rangle = 0$, and so the density correlator depends only on $\boldsymbol{r} - \boldsymbol{r'}$. In the CC phase $\langle S^z\rangle = 0$, $\langle S^y\rangle \neq 0$, and the relative strength of the two systems of dips varies periodically when one changes the initial point $\boldsymbol{r}$, see Fig. 3. This $\boldsymbol{Q}$-dependent contribution stems from the intra-layer currents $\langle a^\dagger_{\sigma,\boldsymbol{r}} a_{\sigma,\boldsymbol{r'}}\rangle = (-)^\sigma (-)^{\boldsymbol{r}} \delta_{\langle \boldsymbol{r}\boldsymbol{r'}\rangle} i\langle S^y\rangle/4$, where $\frac{1}{4}$ comes from the fact that the inter-layer current splits into four equivalent intra-layer ones (see Fig. 1), $\delta_{\langle \boldsymbol{r}\boldsymbol{r'}\rangle}$ means $\boldsymbol{r}$ and $\boldsymbol{r'}$ must be nearest neighbors, and $(-)^{\boldsymbol{r}} \equiv e^{i\boldsymbol{Q}_B\cdot\boldsymbol{r}}$ denotes an oscillating factor. If the correlator is averaged over the particle positions, the CC and DW phases become indistinguishable. A direct way to observe the CC phase could be to use the laser-induced fluorescence spectroscopy to detect the Doppler line splitting proportional to the current.

Bosonic models.– Consider the bosonic version of the model (1), with the additional on-site repulsion U. The effective Hamiltonian has the form (3) with $J = -4(t')^2/V$ and $J_z = \widetilde{V}' + 4(t')^2(1/V - 1/U)$. Due to ferromagnetic (FM) transverse exchange, instead of spontaneous current one obtains the usual Mott phase. CC states can be induced by artificial gauge fields [27]: The vector potential $\boldsymbol{A}(x) = \frac{\pi}{\ell_\parallel}(x + 1/2)$ makes hopping along the x axis imaginary, $t' \mapsto it'$. The unitary transformation $S^{x,y}_{\boldsymbol{r}} \mapsto (-)^{\boldsymbol{r}} S^{x,y}_{\boldsymbol{r}}$ maps the system onto a set of FM chains along the x axis, AF-coupled in the y direction and subject to a staggered field $H = 2t$ along the x axis in the easy (xy) plane. In the ground state net chain moments are arranged in a staggered way along the y axis, so a current pattern similar to that of Fig. 1 emerges, now staggered along only one of the two in-plane directions.

A different type of CC states, with orbital currents localized at lattice sites, can be achieved with p-band bosons [28].

Yet another way to create a CC state in a bosonic bilayer is to introduce the ring exchange on vertical plaquettes:

$$\mathcal{H}_{\mathrm{ring}} = \frac{1}{2}K \sum_{\langle \boldsymbol{r}\boldsymbol{r'}\rangle} (b^\dagger_{1,\boldsymbol{r}} b^\dagger_{2,\boldsymbol{r'}} b_{2,\boldsymbol{r}} b_{1,\boldsymbol{r'}} + \mathrm{h.c.}). \qquad (8)$$

In pseudospin language, the ring interaction modifies the transverse exchange, $J \mapsto J + K$, so for $K > 0$ one can achieve the conditions $J > 0$, $J > |J_z|$ necessary for the CC phase to exist. However, engineering a sizeable ring exchange in bosonic systems is difficult (see [29] for recent proposals).

Spin-$\frac{1}{2}$ bilayer with four-spin ring exchange.– Consider the Hubbard model for spinful fermions on a bilayer shown in Fig. 1, with the on-site repulsion U and inter- and intra-layer hoppings t and t', respectively. At half filling (i.e., two fermions per dimer), one can effectively describe the system in terms of spin degrees of freedom represented by the operators $\boldsymbol{S} = \frac{1}{2} a^\dagger_\alpha \boldsymbol{\sigma}_{\alpha\beta} a_\beta$. The leading term in t/U yields the AF Heisenberg model with the nearest-neighbor exchange constants $J_\perp = 4t^2/U$ (inter-layer) and $J_\parallel = 4(t')^2/U$ (intra-layer), while the next term, with the interaction strength $J_4 \simeq 10t^2(t')^2/U^3$, corresponds to the ring exchange [30, 31]:

$$\begin{aligned}\mathcal{H}_4 &= 2J_4 \sum_{\square} \big\{(\boldsymbol{S}_1\cdot\boldsymbol{S}_2)(\boldsymbol{S}_{1'}\cdot\boldsymbol{S}_{2'}) \\ &+ (\boldsymbol{S}_1\cdot\boldsymbol{S}_{1'})(\boldsymbol{S}_2\cdot\boldsymbol{S}_{2'}) - (\boldsymbol{S}_1\cdot\boldsymbol{S}_{2'})(\boldsymbol{S}_2\cdot\boldsymbol{S}_{1'})\big\}, \quad (9)\end{aligned}$$

where the sum is over vertical plaquettes only (the interaction for intra-layer plaquettes is of the order of $(t')^4/U^3$ and is neglected), and the sites $(1, 2, 2', 1')$ form a plaquette (traversed counter-clockwise). In the same order of the perturbation theory, the nearest-neighbor exchange constants get corrections,

$$J_\perp \mapsto J_R = J_\perp + J_4, \quad J_\parallel \mapsto J_L = J_\parallel + J_4/2,$$

and the interaction $J_D = \frac{1}{2}J_4$ along the diagonals of vertical plaquettes is generated. Generalization for any 2d bipartite lattice built of vertically arranged dimers is trivial. Since $J_\perp \gg J_\parallel, J_4$, we can treat the system as a set of weakly coupled spin dimers. The dynamics can be described with the help of the effective field theory [32] which is a continuum version of the bond boson approach [33] and is based on dimer coherent states $|\boldsymbol{u},\boldsymbol{v}\rangle = (1 - u^2 - v^2)|s\rangle + \sum_j (u_j + iv_j)|t_j\rangle$. Here $|s\rangle$ and $|t_j\rangle$, $j = (x, y, z)$ are the singlet and triplet states, and $\boldsymbol{u}$, $\boldsymbol{v}$ are real vectors related to the staggered magnetization $\langle \boldsymbol{S}_1 - \boldsymbol{S}_2\rangle = 2\boldsymbol{u}(1 - \boldsymbol{u}^2 - \boldsymbol{v}^2)^{1/2}$ and *vector chirality* $\langle \boldsymbol{S}_1 \times \boldsymbol{S}_2\rangle = \boldsymbol{v}(1 - \boldsymbol{u}^2 - \boldsymbol{v}^2)^{1/2}$ of the dimer. Using the ansatz $\boldsymbol{u}(\boldsymbol{r}) = (-)^{\boldsymbol{r}}\boldsymbol{\varphi}(\boldsymbol{r})$, $\boldsymbol{v}(\boldsymbol{r}) = (-)^{\boldsymbol{r}}\boldsymbol{\chi}(\boldsymbol{r})$, passing to the continuum in the coherent states path integral, and retaining up to quartic terms in $\boldsymbol{u}$, $\boldsymbol{v}$, one obtains the Euclidean action

$$\begin{aligned}\mathcal{A} &= \int d\tau d^2r \Big\{\hbar(\boldsymbol{\varphi}\cdot\partial_\tau\boldsymbol{\chi} - \boldsymbol{\chi}\cdot\partial_\tau\boldsymbol{\varphi}) \qquad (10)\\ &+ (\boldsymbol{\varphi}^2 + \boldsymbol{\chi}^2)(J_R - 3ZJ_4/2) - Z[J_\parallel\boldsymbol{\varphi}^2 + J_4\boldsymbol{\chi}^2] \\ &+ (Z/2)[J_\parallel(\partial_k\boldsymbol{\varphi})^2 + J_4(\partial_k\boldsymbol{\chi})^2] + ZU_4(\boldsymbol{\varphi},\boldsymbol{\chi})\Big\},\end{aligned}$$

where the quartic potential U_4 has the form

$$\begin{aligned}U_4 &= (\boldsymbol{\varphi}^2 + \boldsymbol{\chi}^2)[J_\parallel\boldsymbol{\varphi}^2 + J_4\boldsymbol{\chi}^2]/2 \\ &+ J_4(\boldsymbol{\varphi}^2 + \boldsymbol{\chi}^2)^2 + (J_\parallel + J_4)(\boldsymbol{\varphi}\times\boldsymbol{\chi})^2. \quad (11)\end{aligned}$$

Interdimer interactions $J_\parallel$ and J_4 favor two competing types of order: while $J_\parallel$ tends to establish the AF order ($\boldsymbol{\varphi} \neq 0$,

$\boldsymbol{\chi} = 0$), strong ring exchange J_4 favors another solution with $\boldsymbol{\varphi} = 0$, $\boldsymbol{\chi} \neq 0$, describing the state with a *staggered vector chirality*. It wins over the AF one for $J_4 > J_\parallel$, $J_4 > \frac{2}{5Z} J_R$, which for the square lattice ($Z = 4$) translates into

$$J_4 > \max(J_\parallel, J_\perp/9). \tag{12}$$

On the line $J_4 = J_\parallel$ the symmetry is enhanced from $\mathrm{SU}(2)$ to $\mathrm{SU}(2) \times \mathrm{U}(1)$, and the AF and chiral orders can coexist: a rotation $(\boldsymbol{\varphi} + i\boldsymbol{\chi}) \mapsto (\boldsymbol{\varphi} + i\boldsymbol{\chi})e^{i\alpha}$ leaves the action invariant.

The chiral state may be viewed as an analog of the circulating current state considered above: in terms of the original fermions of the Hubbard model, the z-component of the chirality $(\boldsymbol{S}_1 \times \boldsymbol{S}_2)_z = \frac{i}{2}\big\{(a^\dagger_{1\downarrow} a_{2\downarrow})(a^\dagger_{2\uparrow} a_{1\uparrow}) - (a^\dagger_{2\downarrow} a_{1\downarrow})(a^\dagger_{1\uparrow} a_{2\uparrow})\big\}$ corresponds to the *spin current* (particles with up and down spins moving in opposite directions).

Summary.– I have considered fermionic and bosonic models on a bilayer optical lattice which exhibit a phase transition into a circulating current state with spontaneously broken time reversal symmetry. The simplest of those models includes just nearest-neighbor interactions and hoppings, and can possibly be realized with the help of polar molecules.

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